Annual solar motion and spy satellites

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A topic often taught in introductory astronomy courses is the changing position of the Sun in the sky as a function of time of day and season. The relevance and importance of this motion is explained in the context of seasons and the impact it has on human activities such as agriculture. The geometry of the observed motion in the sky is usually reduced to graphical representations and visualizations that can be difficult to render and grasp. Sometimes students are asked to observe the Sun's changing motion and record their data, but this is a long-term project requiring several months to complete. This paper outlines an activity for introductory astronomy students that takes a modern approach to this topic, namely determining the Sun's location in the sky on a given date through the analysis of satellite photography of the Earth.

I. INTRODUCTION

Introductory astronomy courses often devote an early portion of the semester to learning about the motions of celestial objects in the sky, and how those motions change over the course of the year. The apparent North-South motion of the Sun during the year is one of the standard topics used to frame discussions about seasonal changes in the sky.

The seasonal transit of the Sun in the sky, between its northernmost declination on the summer solstice and its southernmost declination on the winter solstice, is often explored in a framework that appeals to our distant ancestors' needs with regards to the agricultural growing season. But in a modern world, where common calendrical systems are used, time and weather are delivered with a quick glance at a smartphone, and few if any of us look beyond the local grocery store for vegetables and other agricultural products, the need to understand the motion of the Sun in the sky seems remote and removed from student lives. Other motivations for understanding the Sun's motion in the Earth's sky, such as building sundials, or explaining the changing energy flux with the seasons owing to the obliquity with which the Sun's rays strike the surface of the Earth can be equally unsatisfying.

Like many topics in astronomy, understanding the motion of the Sun is a highly visual exercise, and observing it in practice takes long term commitments to observations, often over timescales longer than those in which a class is taught. Here we outline an exercise related to the Sun's motion that utilizes a new resource that most students are familiar with – the widespread availability of global satellite imagery through mapping websites and mapping applications.

Most students have found the satellite images of their homes and schools, and tried to guess when the image was taken on the basis of which cars are parked in the driveway or what has been planted in their yards. Here we describe an exercise, based on understanding the motion of the Sun, that allows students to estimate the date on which a given satellite image was acquired. In section II we review the fundamentals of the motion of the Sun; in sections III and IV we describe the process of measurement, date estimation, and error assessment; an example is outlined in section V, and a brief discussion of sources of error in section VI.

Unless otherwise noted, SI units are used throughout.

II. THE MOTION OF THE SUN

The apparent motion of the Sun in the sky results from the combination of two separate motions of the Earth: the daily spin of the Earth on its axis, and the annual revolution of the Earth around the Sun while keeping its angular momentum vector fixed in orientation (pointing very nearly to the star Polaris in the sky).

The Earth's spin produces the daily east-to-west motion of the Sun in the sky. The time for the Sun to move from the meridian that passes over an observer's head (defined to be "local noon") around the sky and back again is called a solar day, and has a length of $t_\odot = 24^h$. This is slightly longer than the Earth's rotational period ($t_\oplus = 23^h 56^m$, called a sidereal day).

Over the course of the year, the Sun appears to move northward and southward in the sky, a long-term observed motion that results from the north pole of the Earth being tipped toward or away from the Sun as the Earth moves in its yearly orbit. The length of time for this motion is known as the *solar year* or *tropical year*, and has a value of $t_{yr} = 365.24$ days.

For the purposes of the exercise described here, the most convenient sky coordinates to describe the location of the Sun are declination (coordinates lines equivalent to the Earth's latitude lines projected onto the sky) and meridians (coordinate lines fixed on the sky, but parallel to the projected lines of the Earth's longitude).

FIG. 1: Different solar tracks across the sky as a function of the time of year, as viewed by a northern hemisphere observer standing facing south.

FIG. 2: A satellite view of New York City retrieved from Google Maps. Differences in shadow directions across the image show the mosaic was constructed by data taken at different times.

Representative daily solar tracks across the sky for observers in the Northern Hemisphere are shown in Figure 1 for different times of year. During the northern summer, the Sun's position has the following properties: it is at high declination; it reaches higher altitudes in the sky; it rises and sets north of the east and west cardinal directions; and it is visible for more than 12 hours a day. During the northern winter, the Sun's position has the properties: it is at lower declination; it reaches lower maximum altitude during the day; it rises and sets over the horizon at points south of the cardinal east and west directions; and it has total visibility of less than 12 hours per day. The principal effect of the slow change in solar declination is the increase and decrease of flux (power per unit area) on the surface of the Earth as the obliquity of the Sun's rays changes; this is the dominant physical effect that drives the change in the seasons. The obliquity of the Sun's rays and the geometrical consequences for the projection of shadows is precisely the effect that this activity exploits.

III. MEASURING SATELLITE IMAGES

A. Target selection

Most online mapping services, as well as geographical mapping applications like `GoogleEarth` have an information layer comprised of satellite imagery that has been stitched together into a continuous mosaic across the surface of the Earth. A variety of visual indicators show that the mosaics have been constructed from images taken at different epochs. For instance, most of the areas are virtually cloudless, and the seasons indicated by vegetative states and ground cover seen from one area of the map to another are not always constant. A prominent indicator is the differences in shadows from one side of an image to another. Consider Figure 2, showing a piece of a satellite mosaic of New York City. On the left side of the image, the shadows clearly point in a different direction than those on the right hand side of an image. When were these two different images taken? Students are motivated by the same question when viewing areas that are familiar and important to them.

In the activity described here, students should pick a target they are familiar with — for example their homes, a public school, or a local library. The procedure described in this paper can be completed for *any* satellite image so long as three basic requirements are met: (1) The image must show the shadow of an object on the ground; (2) The geographic location of the object must be known (*i.e.*, latitude and longitude); (3) The dimensions of the object (height, width and length), and its orientation with respect to the compass points must be known. The height h is measured from the ground level to the piece of the structure that corresponds to the outermost part of the shadow in the satellite image.

For the purposes of illustration in this article, consider a well known national landmark for which the physical data can be easily found: the Washington Monument in Washington D.C. A retrieved image of the Washington Monument is shown in Figure 3. The Washington Monument is an excellent target for demonstration purposes, as its location and dimensions are well known.

B. Measuring shadows and angles

The Sun traces a regular pattern in the sky as a function of time, a pattern that is captured in the length and direction of shadows on the ground. To reconstruct the position of the Sun in the sky, the students need to measure two basic pieces of information from a satellite image: (1) the length of the shadow s, which is converted into the solar *altitude angle* $\mathcal{A}$ (angle of the Sun above the horizon), and (2) the direction the shadow points, which is converted into the solar *azimuth angle* $\mathcal{Z}$, measured from due north clockwise toward the east.

The most convenient method of making measurements off of satellite images is to import a screen capture into a graphics program. Using the mouse, have students identify a pixel at the base of the shadow, and record the cartesian location of the pixel, (x_1, y_1), the graphics software reports for the pixel. Do the same for a second pixel at the apex of the shadow, recording its cartesian loca-

FIG. 3: A satellite image of the Washington Monument, retrieved from Google Maps.

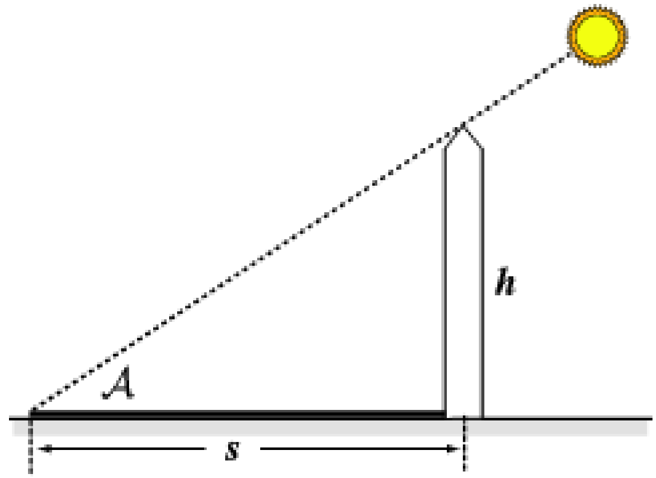

FIG. 4: The solar altitude angle $\mathcal{A}$ is derived from the shadow length s and target height h.

tion, (x_2, y_2). The length s of the shadow is found by application of the Pythagorean theorem,

$$s = \sigma\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \ , \tag{1}$$

where σ is the conversion scale in units of distance/pixel. With the known height h of the target casting the shadow, the length of the shadow gives the altitude angle of the Sun from the geometrical construction shown in Figure 4. The result is

$$\mathcal{A} = \tan^{-1}\left(\frac{h}{s}\right) \ . \tag{2}$$

The azimuth angle $\mathcal{Z}$ is defined as the angular displacement of the Sun, measuring parallel to the horizon from due north eastward. The angular location of the shadow, C, is measured from due north, and may be measured directly from the image using a protractor or computed using the cartesian coordinates for the end points of the shadow. If Δx and Δy are the coordinates offsets between the two ends of the shadow, then

$$C = \tan^{-1}\left(\frac{|\Delta x|}{|\Delta y|}\right) \ . \tag{3}$$

The Sun is located exactly opposite the shadow, so the azimuth angle is

$$\mathcal{Z} = 180^{\circ} \pm C \ . \tag{4}$$

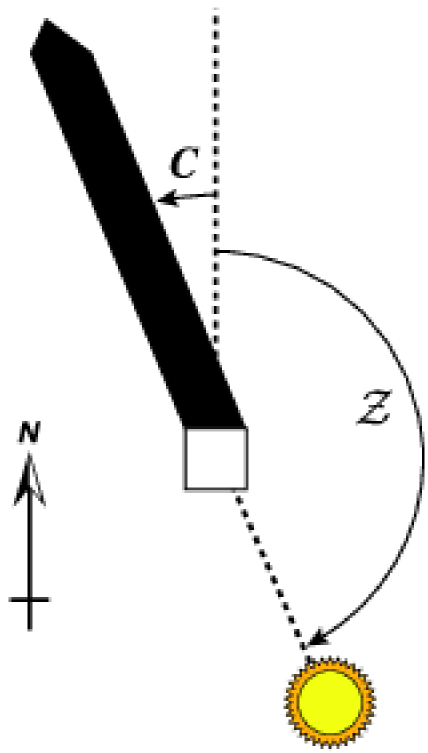

FIG. 5: The shadow angle C and the solar azimuth angle, $\mathcal{Z}$. It is convenient to work in coordinates aligned to the cardinal directions, such that the y axis is aligned north-south, and the x axis is aligned east-west.

The sign in Eq. 4 is chosen to give the correct value as measured from north, and depends on the orientation of the shadow in the satellite image.

Because the rotations of the sky are fixed around the north celestial pole (near the star Polaris), the reconstruction of the date and time from solar position is most conveniently carried out in a spherical coordinate system tied to the pole known as *equatorial coordinates*: a lati-

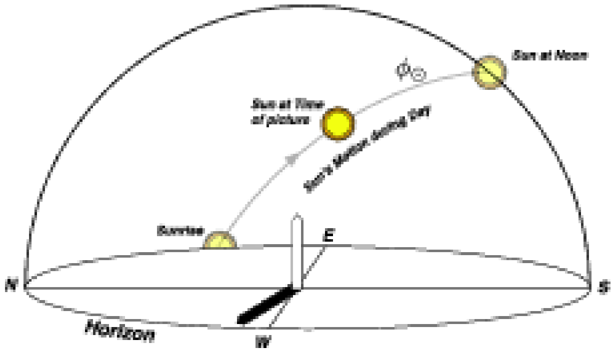

FIG. 6: The solar meridian $\phi_\odot$ is the angle between the location of the Sun and the central meridian over the landmark, measured along the Sun's path.

tude like angle known as declination, δ, and a longitude like angle known as right ascension, α. Many formulae exist for converting $(\mathcal{A}, \mathcal{Z}) \rightarrow (\delta, \alpha)$ [1, 2], but most depend on knowing the observer's local time, which is *unknown* in this activity (it is the time the photograph was taken). Instead, the conversion may be derived generically in terms of a coordinate rotation at a fixed time; the offset of the Sun from the fixed value then is a measure of the true clock time at which the image was taken. The coordinate rotation is not germane to the student's understanding of this activity and has been omitted here, but the result of such a calculation is the solar declination,

$$\delta_\odot = \sin^{-1}\left[\sin\mathcal{A}\cdot\sin L + \cos L\cdot\cos\mathcal{Z}\cos\mathcal{A}\right] \ , \qquad (5)$$

where L is the latitude of the target in the satellite image. This is the true value of the solar declination at the time the satellite image was taken, and is strictly a function of the date, as will be seen in the next section. Note that astronomical declinations are positive for angles north of the equator, and negative for angles south of the equator.

Meridians are fixed lines of longitude on the sky, running from the north celestial pole to the south celestial pole. Define a solar meridian $\phi_\odot$ to be the angle measured from the meridian that passes directly overhead to the meridian that passes through the Sun at the time of the satellite photograph, as illustrated in Figure 6. In terms of the known observables, the solar meridian is given by

$$\phi_\odot = \tan^{-1}\left(\frac{\sin\mathcal{Z}\cdot\cos\mathcal{A}}{\sin\mathcal{A}\cdot\cos L - \sin L\cdot\cos\mathcal{Z}\cdot\cos\mathcal{A}}\right) \ . \qquad (6)$$

IV. RECOVERING DATE AND TIME

A. Time Reconstruction

On any given day, the Sun makes one transit around the celestial pole from the vantage point of Earth in a time $t_\odot$. It crosses the observer's local meridian (the line running from due north, overhead to due south) at precisely noon local time. The Earth rotates with a sidereal period $t_\oplus$ giving a rotational frequency $\omega_\oplus = 2\pi/t_\oplus$. The time it takes the Sun to traverse the angle $\phi_\odot$ then is

$$t_\phi = \frac{\phi_\odot}{\omega_\oplus} = \frac{\phi_\odot\cdot t_\oplus}{2\pi} \ . \qquad (7)$$

The time t_ϕ is the time of the photograph relative to local noon. Note that local noon is defined by the time the Sun is on the local meridian, *not* by the time in your current time zone. Values of $t_\phi > 0$ indicate times before local noon, whereas $t_\phi < 0$ are times after local noon.

The time reference on Earth is known as Coordinated Universal Time (UTC), with the origin defined at midnight on the prime meridian. The local time expressed in UTC depends on the longitude β as $t_{UTC} = t_{local} - \beta/\omega_\oplus$, where t_{local} is the time of interest, in this case the time the satellite images was taken. For this offset to work properly at any longitude on Earth, eastern longitude angles are taken to be positive and western longitude angles are taken to be negative. For this calculation, the point of reference is local noon, so choose $t_{local} = t_N - t_\phi$, where t_ϕ is the offset from noon described above, and t_N is a constant equivalent to 12 hours in the units of choice. Overall then, the time a given photograph was taken, in UTC is

$$t_{UTC} = (t_N - t_\phi) - \frac{\beta}{\omega_\oplus} = t_N - \frac{1}{\omega_\oplus}\left(\phi_\odot + \beta\right) \ . \qquad (8)$$

B. Date Reconstruction

The date is established by the declination of the Sun, $\delta_\odot$, at the time the satellite image was taken. A variety of analytic formulae exist for the declination of the Sun as a function of the day of the year, D. A simple formula that can easily be manipulated algebraically is [3]

$$\delta_\odot = 23.44^\circ\cdot\sin\left[\frac{2\pi}{365}(D-81)\right] \ . \qquad (9)$$

This can be inverted for day of the year, D:

$$D_1 = 81 + \frac{365}{2\pi}\cdot\sin^{-1}\left(\frac{\delta_\odot}{23.44^\circ}\right) \qquad (10)$$

or

$$D_2 = 81 + \frac{365}{2\pi}\left[\pi - \sin^{-1}\left(\frac{\delta_\odot}{23.44^\circ}\right)\right] \qquad (11)$$

where $\delta_\odot$ is expressed in degrees. Note that the arcsine terms should not be blindly taken with a calculator; the result of $sin^{-1}(\delta_\odot/23.44^\circ)$ should be in *radians*!

Why are there two solutions? The Sun's annual motion crosses a particular declination in the sky twice each year — once when it is heading northward through the sky,

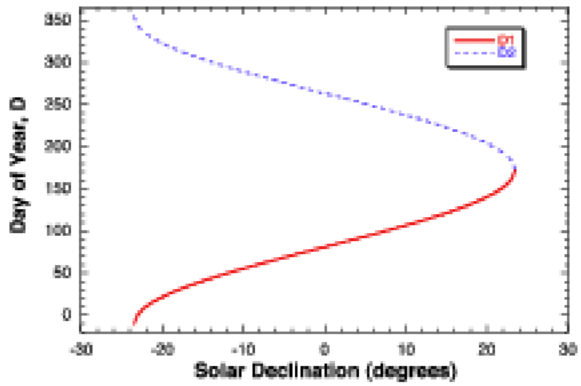

FIG. 7: The complete solution for day of the year (D_1 or D_2) as a function of the solar declination, $\delta_\odot$.

FIG. 8: A simulated solar analemma for 4:20pm local time at mid-northern latitudes. The sun traces out the figure over the course of the year, crossing each declination twice.

and once when it is heading southward. Figure 7 shows the two branches covered by D_1 and D_2. The most clear observational demonstration of this is the photographic capture of the analemma, the Sun's apparent motion in the sky when photographed at the same time every day. A simulation of the analemma from a desktop planetarium program is shown in Figure 8. When students are attempting to decide which solution to use to determine the day of the year, they must rely on auxiliary data from the photograph, such as vegetative cover or other seasonal clues.

V. EXAMPLE: THE WASHINGTON MONUMENT

It is useful to examine a concrete example of the method outlined above. The initial specifics of the image scale σ and the shadow measurements Δx and Δy will depend on the captured image used, but the other details should be similar to the values derived here.

The satellite image of the Washington Monument previously shown in Figure 3 has the necessary qualities for this activity. The geographic location of the Monument is well known (LAT = 38.889463° N, LONG = 77.035242°W), as are its dimensions (height $h = 169.294$ m, square base with sides of length $\ell = 16.80$ m). Furthermore, it is oriented such that the faces are parallel to the cardinal directions.

Using the satellite image of the Monument, the image is loaded into a graphics program that will report the coordinates of any pixel in the image. The scale was measured to be $\sigma = 0.469$ meters/pixel. Using a one pixel coordinate at the center of the Monument for one end of the shadow, and the pointed tip of the shadow for the other end, we obtained $\Delta x = 86$ pixels and $\Delta y = 213$ pixels, giving a shadow length of $s = 107.7$ meters and a solar azimuth angle of $\mathcal{Z} = 202.0°$. Using this shadow length with the known height of the Monument, the solar altitude angle was found to be $\mathcal{A} = 57.5°$. Converting $\mathcal{A}$ and $\mathcal{Z}$ to solar declination and solar meridian using Eqs. 5 and 6 with the known geographical coordinates of the Monument yields $\delta_\odot = 8.18°$ and $\phi_\odot = -11.7°$.

Reconstructing the time from Eq. 8 yields $t = 17.9$ h UTC, which in Eastern Daylight Time is EDT = UTC − 4 h = 13.9 h = 1:54 PM. The dates are $D_1 = 101.7 \simeq 102$ (= April 12) and $D_2 = 242.8 \simeq 243$ (= August 31).

VI. ERRORS AND ANALYSIS

This activity provides an excellent opportunity to introduce students to the concepts of error analysis. In our experience, the most common source of error is in determining the precise length of the shadow. The shadow length depends on where the endpoints of the shadow are chosen, and also on the measurement of the image conversion scale σ. Error from these measurements can be calculated from Eqs. 1 and 3 using standard techniques [5], and propagated to the equations for solar declination (Eq. 5), solar meridian (Eq. 6), and ultimately the time (Eq. 8) and date (Eqs. 10 and 11).

To minimize the error in σ, students can estimate the image scale by measuring the length of a known object visible in a satellite image, then counting the equivalent pixel coverage in the satellite image. A student can use the length of their house, or a sidewalk located in the satellite image close to the shadow of interest. For objects not co-aligned with the pixel grid, they will have to apply the Pythagorean theorem to determine the length of the object in pixels, just as they do for the shadow. This method of direct image scale determination will yield a more accurate value for the conversion scale σ, minimizing the overall error.

Absolute errors may be determined by taking a student solution for a particular date and time, entering them into a desktop planetarium program, then comparing the Sun's reported altitude $\mathcal{A}$ and azimuth $\mathcal{Z}$ to the student values derived from Eqs. 2 and 4.

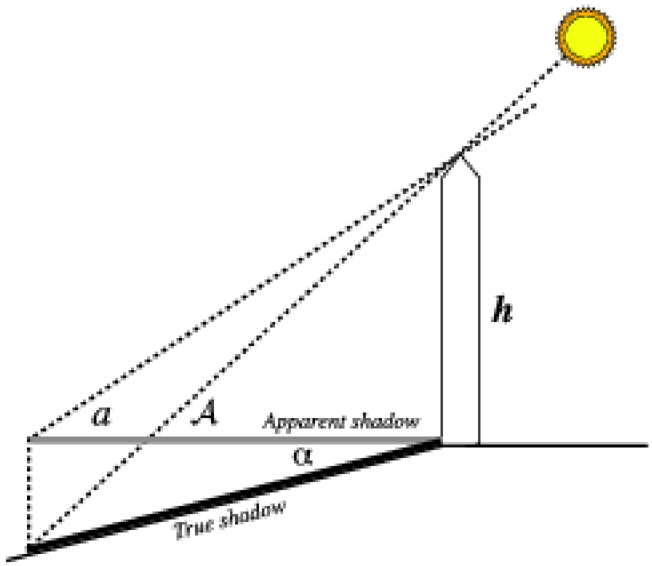

FIG. 9: When the shadow is projected on a topographic slope of angle α, the apparent shadow length ℓ measured from an overhead satellite photograph is different than the true shadow length. The result is that the apparent solar altitude angle a will be different than the true solar altitude angle $\mathcal{A}$.

One of the most significant sources of errors is when shadows are projected onto slopes. Figure 9 illustrates the case for a shadow projected downslope, which results in an apparent shadow length ℓ different than the true shadow length. If the apparent shadow length is used in Eq. 2, the recovered solar altitude angle will not be the true value. If the slope α is known (which students can measure from topographical maps, or measure with rudimentary surveys), then the true solar altitude $\mathcal{A}$ may be found from the measured apparent shadow length ℓ in the photograph, the object height h and the slope angle α using some basic geometry:

$$\tan \mathcal{A} = \frac{h}{\ell} + \tan\alpha \ . \qquad (12)$$

VII. DISCUSSION

Making science relevant to students and their lives is always a challenge, particularly in service level courses. In courses like introductory astronomy, the task is even more important since astronomy will for many students be the only science class they ever take.

The strengths of this activity are that: (1) it builds on everyday, common experiences that most students have, namely looking at their homes using online satellite images; (2) it addresses a basic concept that is regularly taught as part of the introductory astronomy repertoire; and (3) it is adaptable to any location on the globe. It is somewhat mathematical in derivation, but not outside the norm of what is often expected of introductory astronomy students. It is, however, amenable to "measure, plug and play" type laboratory exercises that are common in such courses, reducing the work to making direct measurements that are then evaluated using algebraic formulae.

An ideal follow-on or companion activity would be to work with a set of satellite images of the same area taken at different times, to evaluate the changing position of the Sun, or to monitor the Sun's declination over a long period of time (a common long term observing activity in many astronomy courses).

A version of this activity suitable for the classroom has been written up and published in the Teaching section of the *Instructables* website. You can find it at `www.instructables.com/id/Time-and-Date-of-Satellite-Maps/`

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