DATING SATELLITE PHOTOGRAPHS
Equation Reference Sheet

This quick reference sheet is provided to show the mathematical equations for the reconstruction of the date and time based on shadow observations in satellite photos. They are keyed to particular STEP numbers in the Instructable, which has complete details.

Reference table of symbols used

Symbol	Identity	Units
lat	Latitude of landmark	decimal deg
lng	Longitude of landmark	decimal deg
B	Baseline - distance from landmark when measuring its height	meters
a	altitude - measured angle to top of landmark with theodolite	degrees
h	computed height of landmark from measurements	meters
K	Map Scale, measured from known object on satellite image	meters/pixel meters/cm
L	Shadow length measured on image	pixels or cm
s	True shadow length computed from image and map scale	meters
$\mathcal{A}$	Solar altitude, computed from shadow length and landmark height	degrees
$\mathcal{Z}$	Solar azimuth, computed from shadow direction	degrees
$\phi_\odot$	Solar meridian (in text written PHI)	degrees
$\delta_\odot$	Solar declination (in text written DEC)	degrees
D_1, D_2	Day of year	day

Equations Used by Step

► **Step 4** From a measured baseline B and a measured angle a, the height of the landmark is:

$$h = B \cdot \tan(a) \ . \tag{1}$$

► **Step 6** From the landmark height h and the shadow length s the solar altitude $\mathcal{A}$ is

$$\mathcal{A} = \tan^{-1}\left(\frac{h}{s}\right) \ . \tag{2}$$

► **Step 7** In terms of the known observables, the solar meridian is given by

$$\phi_\odot = \tan^{-1}\left(\frac{\sin(\mathcal{Z}) \cdot \cos(\mathcal{A})}{\sin(\mathcal{A}) \cdot \cos(lat) - \sin(lat) \cdot \cos(\mathcal{Z}) \cdot \cos(\mathcal{A})}\right) \ . \tag{3}$$

► **Step 7** The time reference on Earth is known as Coordinated Universal Time (UTC), with the origin defined at midnight on the prime meridian. The time a given photograph was taken, in UTC is

$$UTC = 12h - \frac{\phi_\odot + lng}{15.04178} \ . \tag{4}$$

► **Step 8** Given the latitude lat, the solar altitude $\mathcal{A}$ and the solar azimuth $\mathcal{Z}$, the solar declination is given by

$$\delta_\odot = \sin^{-1}\left[\sin(\mathcal{A}) \cdot \sin(lat) + \cos(lat) \cdot \cos(\mathcal{Z}) \cos(\mathcal{A})\right] \ , \tag{5}$$

Note that astronomical declinations are positive for angles north of the equator, and negative for angles south of the equator.

► **Step 8** The date is established by the declination of the Sun, $\delta_\odot$, at the time the satellite image was taken. The day of the year is:

$$D_1 = 81 + \frac{365}{360} \cdot \sin^{-1}\left(\frac{\delta_\odot}{23.44^\circ}\right) \tag{6}$$

or

$$D_2 = 81 + \frac{365}{360}\left[180 - \sin^{-1}\left(\frac{\delta_\odot}{23.44^\circ}\right)\right] \tag{7}$$

Here, make sure the Arcsin functions return the answer in *degrees* not *radians*, otherwise these forms of the equations will not give the correct day.