TAM 335 Lab 1 Report:
Elementary Laboratory Procedures

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Introduction

The purpose of this lab was to measure steady state volumetric flow and pressure differences. Specifically, the weight time method was applied to measure flow rates. Pressure differences were calculated two different ways, using Bourdon pressure gages and a differential manometer. The benefits and trade-offs of using the Bourdon pressure gages vs. the differential manometer were considered. Finally, the relationship between steady state volumetric flow rate and pressure difference was demonstrated and explored.

The weight-time method was used to measure the steady volumetric flow rate. This procedure requires a container to capture the entire flow, a scale to measure the weight W of the fluid, and a stopwatch to record the time Δt it takes to do so. The mass flow rate $\dot{M}$ and volumetric flow rate Q can be defined by:

$$\dot{M} = \frac{W}{g\Delta t} \tag{1}$$

and

$$Q = \frac{\dot{M}}{\rho} = \frac{W}{\rho g \Delta t} = \frac{W}{\gamma \Delta t}, \tag{2}$$

where ρ is the density of the fluid, $\gamma = g\rho$ is the specific weight of the fluid, and g is the gravitational constant.

The volumetric flow rate of water can be determined experimentally by choosing a pre-determined weight W, the specific weight of water γ_w =9.81 kN/m^3, measuring the time Δt it takes for the container to fill up with W lbs. of water, and plugging these values into Eqn. (2).

Two instruments will be used to measure the pressure difference in the pipe with steady state volumetric flow. The apparatus used in this lab to measure the pressure differences and flow rate are shown in Fig. 1 below. Bourdon pressure gauges are a useful tool to calculating pressure difference. Referring to Fig. 1, which shows the apparatus used in this lab for these measurements, the pressure measurements p_1 and p_2 can be read from each Bourdon gage. Taking the pressure reading from the corresponding Bourdon gage and accounting for the additional pressure due to the weight of the water between each gage and point, the pressures at point *A* and point *B* are:

$$p_A = p_1 + \gamma_w(a_1 - a_A) \quad (3)$$

and

$$p_B = p_2 + \gamma_w(b_2 - B), \quad (4)$$

where $(a_1 - a_A)$is the height difference of p_1 and p_A and $(b_2 - B)$ is the height difference between p_2 and p_B, both of which are shown in Fig. 1. Subtracting Eqn. (3) and Eqn. (4), the pressure difference $p_A - p_B$ can then be calculated.

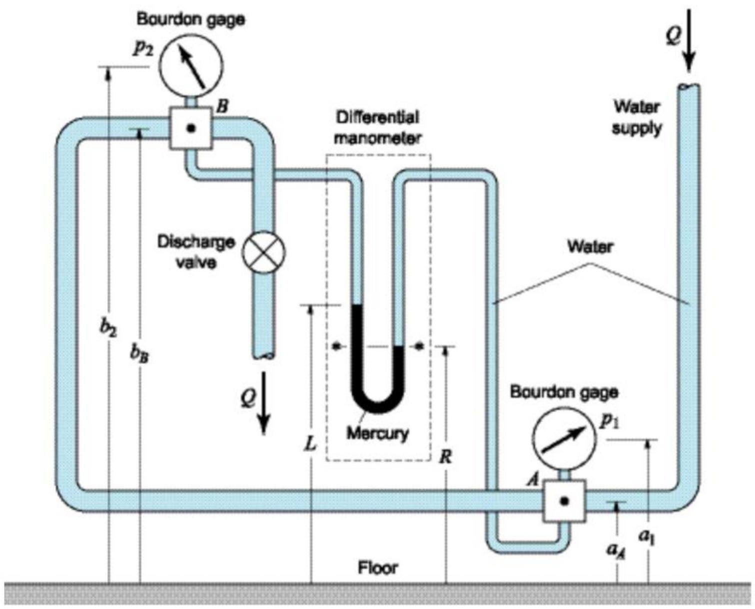

Figure 1. Apparatus used to measure flow rate and pressure differences

Alternatively, $p_A - p_B$ can be determined using the differential manometer. The scale located between the columns of the manometer gives the readings for h_1 and h_2, as shown below in Fig. (2). It should be noted that the sign conventions are always $h_1 > 0$ and $h_2 < 0$ for the manometer measurements. The height difference $\Delta h = h_1 - h_2$ can then be used to find the pressure difference:

$$p_A - p_B = \gamma_w(b_B - a_A) + (\gamma_{Hg} - \gamma_w)(h_L - h_R). \tag{5}$$

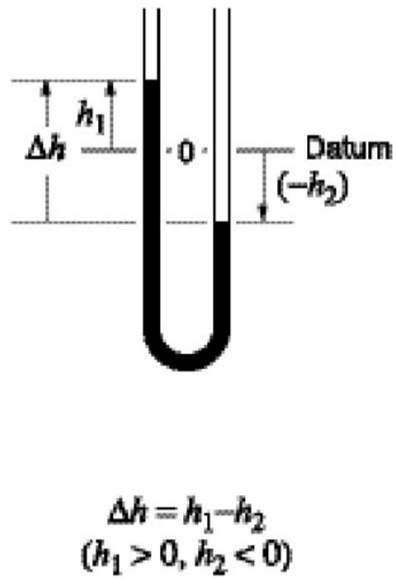

Figure 2. Differential manometer measurements

The concepts discussed above were applied in the laboratory procedure to test these relationships and objectives.

Experimental Methods

In the laboratory, a tank fixed to a beam scale, equipped with a pipe above to control water flow into the tank and a drain to control water flow out of the tank, was the apparatus used to measure the volumetric flow rate. Additionally, two Bourdon gages, as depicted in Fig. (2), are located at heights a_A and b_B and a differential manometer to measure pressure differences.

The first part of the procedure was to establish the maximum water flow Q_{max} from the pipe, with the drain open. Once the flow was constant, the pressure values p_1 and p_2 were recorded from each Bourdon gage and the heights on the differential manometer h_L and h_R were recorded. Next, the drain was closed and the time Δt to fill the tank with W = 100 lbs. of water was measure with a stopwatch. The maximum flow rate Q_{max} was calculated by plugging Δt into Eqn. (2).

Comparing the two techniques for calculating pressure, the Bourdon gages gave pressure readings p_1 and p_2 in units kPa. Equations (3) and (4) were then used to find the corresponding values of p_A and p_B, and their difference was calculated. This differs from how $p_A - p_B$ was calculated using the manometer reading. The scale on the manometer showed the values for h_L and h_R with the units *cm*. The value h_L was positive, and h_R was negative. Referring to Fig. (2), h_L corresponds to h_1, and h_R corresponds to h_2. These values could be substituted into Eqn. (5) to find the alternative $p_A - p_B$ calculation. Finally, $\Delta h_{max} = h_L - h_R$ was computed to be used in the next iterations of this same procedure. It should be noted that the following data was provided for use in the laboratory manual for making these calculations: a_1 = 0.810 m, a_A = 0.613 m, b_2 = 2.376 m, b_B = 2.183 m, γ_w = 62.4 lb / ft^3 = 9810 N / m^3, and γ_{Hg} / γ_w = 13.55.

Once Q_{max} and Δh_{max} were established, the procedure was repeated for slower flow rates. Five additional flows were analyzed: 80% Q_{max}, 60% Q_{max}, 40% Q_{max}, 20% Q_{max}, and 0% Q_{max}. Because the relationship between volumetric flow rate and pressure difference is quadratic, the manometer deflections Δh_{max}, $0.64\Delta h_{max}$, $0.36\Delta h_{max}$, $0.16\Delta h_{max}$, $0.04\Delta h_{max}$ and 0 could be used to achieve each respective flow percentage.

One step in the lab that proved to be challenging was using the scale to measure 100 lbs. of water in the tank for each iteration. To do so, a weight marker that represented 100lbs. of water was placed on the balance pan, causing the beam to fall against the bottom stop. The time it took for the scale to rebalance, indicating that the tank had been filled with 100lbs of water, was then recorded.

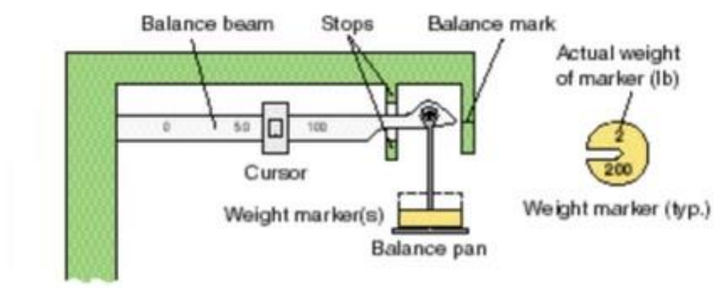

Figure 3. Balance beam of scale

There are a few vulnerable points in the procedure where the possibility for error was present. The first opportunity for error is in calculating Δt. This was done manually with our phones. In effort to minimize error, each participant in our lab group measured Δt, and the average of these values was the value recorded for Δt in each iteration. Another step in the lab, where error seemed likely, was in recording pressure and height values discussed above. We used our best judgement in making these reading, but it is possible some of these readings were inaccurate. Additionally, all iterations of this procedure for 80% Q_{max}, 60% Q_{max}, 40% Q_{max}, 20% Q_{max}, and 0% Q_{max} all depend on the value h_{max} calculated in the first iteration. The results for all smaller flows depend on the quality of data collected in the first iteration for Q_{max}. Finally, it should be mentioned accuracy of measurements improves proportionally to size of tank and Δt. For this lab, we were constrained to a weigh tank with 100 lbs. capacity.

Results and Discussion

	Weight-Time				Bourdon Gages			Manometer		
Reading No	Weight [lbs]	Time [s]	Q [ft³/s]	Q [m³/sec]	p1 [kPa]	p2 [kPa]	pA-pB [kPa]	hL [cm]	hR [cm]	pA-pB [kPa]
1	100	**43.4060**	0.03692034	0.001045606	70	15	55.0392	14	-15.5	51.720625
2	100	54.23	0.02955343	0.00083697	102	58	44.0392	8.6	-10	38.30109
3	100	78.54	0.02040547	0.000577895	118	88	30.0392	4.6	-5.9	28.328775
4	100	124.31	0.01289126	0.000365088	134	110	24.0392	1.75	-3	21.2496625
5	100	257.72	0.00621824	0.000176104	142	123	19.0392	0.2	-1.1	17.002195
6	100		#DIV/0!	#DIV/0!	145	128	17.0392			15.4017

Figure 4. Bordon Gages and Manometer Measurements

Time Readings (Average at the bottom used in table)

1	2	3	4	5	
43.6	54.58	78.26	124.52	255.74	
43.38	53.84	78.17	124.61	255.7	
43.6	54.43	78.89	124.04	255.44	
43.58	54.14	78.68	124.42	266	
42.87	54.14	78.68	123.98	255.72	
43.4060	**54.226**	**78.536**	**124.314**	**257.72**	**#DIV/0!**

Figure 5. Time Readings

The data collected from the lab procedure is shown in Fig. (4) and Fig. (5). These results were used to compare pressure measurement methods, analyze relationship between pressure difference and volumetric flow rate, and estimate the precision of the measurements.

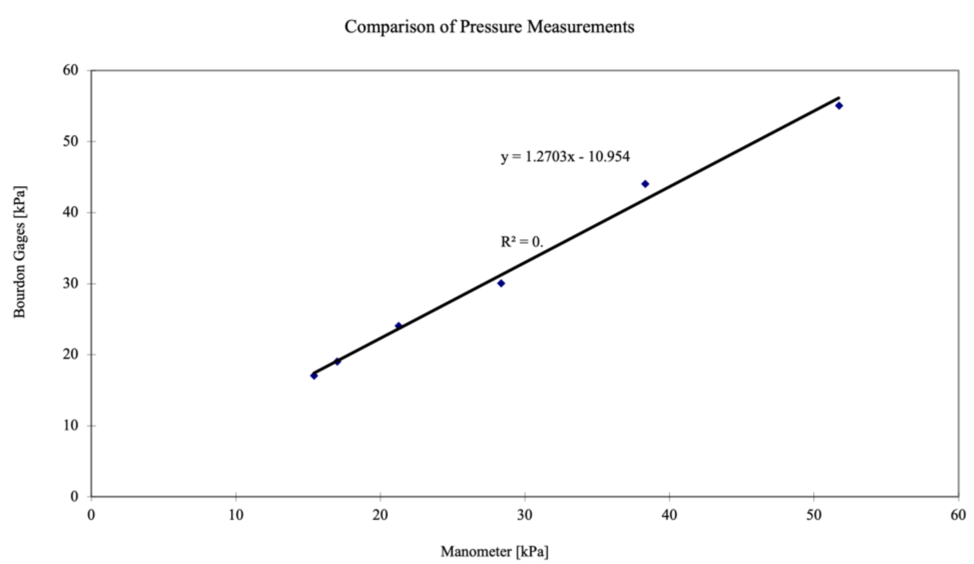

Figure 6. Graph Comparison of Pressure Measurements

To answer Lab Report #1, a graph of the pressure difference $p_A - p_B$ calculated with the Bourdon Gage method as a function of the pressure difference $p_A - p_B$ calculated with the Manometer is plotted as a graph in Fig. (6). The fitted line shows the relationship $y = 1.2703x - 10.954$.

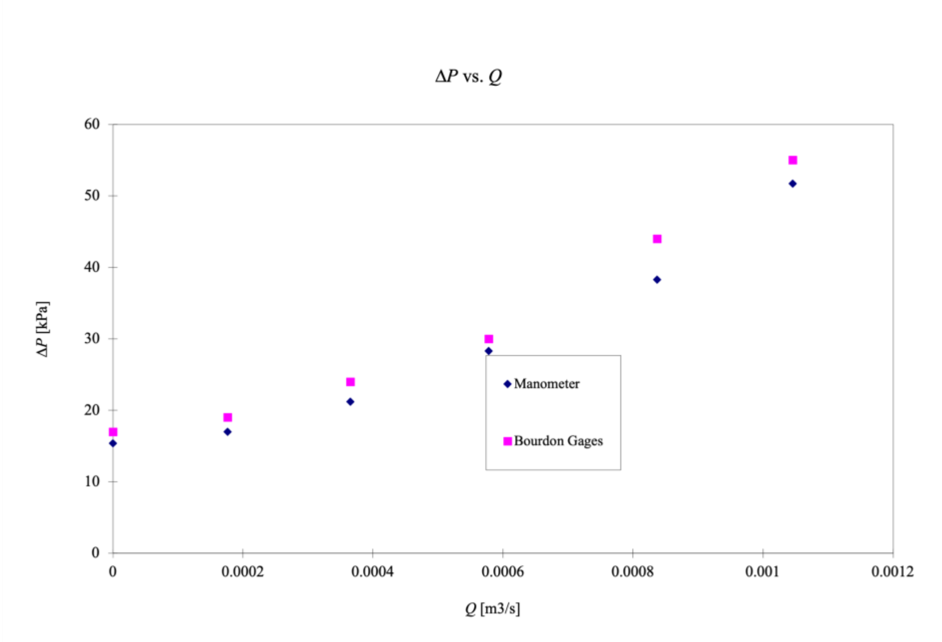

Figure 7. Graph of Δp vs Q

To answer Lab Report #2, the graph in Figure 7 was plotted, showing the Δp in kPa for both methods as a function of flow rate Q in m^3/s. The shape of both functions confirms the assumption made in lab that the pressure difference Δp is a quadratic function of volumetric flow rate Q. There is a y-intercept for both methods, which suggests that measurements were more accurate when flow rate is higher when using this apparatus. The manometer method seems to be more reliable because the measurement Δh was more precise, had a higher accuracy, and took the least amount of time to record. While carrying out the procedure, the Bourdon Gage readings fluctuated, and it was difficult to get an accurate reading at times. This was especially noticeable in the last few iterations, as the water flow was reduced, and the calculated pressure difference was smaller. One possible cause for this is that the Bourdon gage readings were pressure calculations, whereas the differential manometer height difference Δh was a simple measurement.

To answer Question #1, the precision of the measurements taken using the weight-time method were estimated with the following equation:

$$e = \frac{Q_s - Q_l}{\frac{1}{2}(Q_s + Q_l)}, \tag{6}$$

where e is the precision, the Q_l is the flow rate calculated from the shortest time measurement of the first weight-time data set, and Q_l is the flow rate calculated from the longest time measurement of the first weight-time data set. The data in the first column of Fig. (5) depicts the first weigh-time data set. Using the shortest time measurement Δt=42.87 and Eqn. (2), the flow rate calculated from the shortest time measurement is Q_s =0.001058 m^3/s. Similarly, the flow rate using longest time measurement Δt = 43.6 s, was Q_l = 0.00104 m^3/s. Finally, using Eqn. (6), the precision was calculated to get e = 0.0172 = 1.72%. The calculations for Q_1, Q_s, and e can be found in Appendix 1. Typical engineering calculations allow 5% error. The precision e = 1.7% falls within this threshold, so it can be concluded that the measurements have acceptable precision for engineering standards. The first data set was chosen for this analysis because accuracy of measurement is proportional with Δt. Each iteration following the first data collection had increased Δt values, which means they were more precise than the first one data set.

Conclusions and Recommendations

Lab One revealed the quadratic relationship between pressure difference Δp and volumetric flow rate Q. The presence of a y-intercept in experimental analysis of this relationship indicates that pressure differences are easier to resolve when flow rate is higher. The Bourdon gage method and differential manometer method were both used to calculate pressure differences. From experience collecting data in the lab and analysis of both processes, it can be concluded that the differential manometer is more reliable. Additionally, taking these measurements are easier for high volumetric flow rates and pressure differences. In the future, it would be interesting to investigate volumetric flow rates by holding Δt constant, varying the water flow, and measuring the weight W of the total flow in that time period. Additionally, adjusting the apparatus used to measure pressure differences could minimize the y-intercept present in the quadratic relationship between pressure difference Δp and volumetric flow rate Q, improving the accuracy of measurements of pressure differences for smaller flow rates.

References

Lab 1 Manual: Elementary Lab Procedures by Keane and Phillips

Appendix 1

Calculation for Q_s:

$$Q_s = \frac{W}{\gamma \Delta t} = \frac{100\ lbs}{62.4\ lb/ft^3 \times 42.87\ s} \times \frac{0.0283\ m^3/s}{1\ ft^3/s} = 0.001058\ m^3/s$$

Calculation for Q_l:

$$Q_l = \frac{W}{\gamma \Delta t} = \frac{100\ lbs}{62.4\ lb/ft^3 \times 43.6\ s} \times \frac{0.0283\ m^3/s}{1\ ft^3/s} = 0.00104\ m^3/s$$

Calculation for *e*:

$$e = \frac{Q_s - Q_l}{\frac{1}{2}(Q_s + Q_l)} = \frac{0.001058 - 0.00104}{\frac{1}{2}(0.001058 + 0.00104)} = 0.0172 = 1.72\%$$